

Question 8

An elliptic paraboloid has vertex $(1, -2, 3)$, axis parallel to the z -axis, opens downward, and its trace in $z = -1$ is

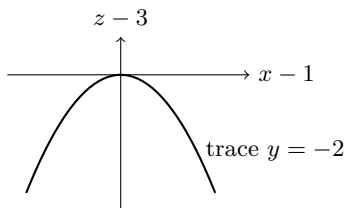
$$\frac{(x-1)^2}{8} + \frac{(y+2)^2}{4} = 1.$$

Tasks

1. Reconstruct the surface equation.
2. Find the trace at $z = 2$ in standard form.
3. State that trace's semiaxes and verify the supplied trace.

Question 8 – Solution

Strategy Use translated form $z - 3 = -A(x - 1)^2 - B(y + 2)^2$ and fit the known level.



Step 1: Use the known trace At $z = -1$, the vertical drop is 4. The given ellipse is equivalent to

$$\frac{1}{2}(x - 1)^2 + (y + 2)^2 = 4.$$

Hence the surface equation is

$$z - 3 = -\frac{1}{2}(x - 1)^2 - (y + 2)^2.$$

Step 2: New trace Set $z = 2$:

$$1 = \frac{1}{2}(x - 1)^2 + (y + 2)^2,$$

or

$$\frac{(x - 1)^2}{2} + (y + 2)^2 = 1.$$

Its semiaxes are $\sqrt{2}$ and 1. Substituting $z = -1$ back into the derived surface recovers the supplied ellipse.