

Question 7

The ellipsoid

$$\frac{x^2}{16} + \frac{y^2}{9} + \frac{z^2}{4} = 1$$

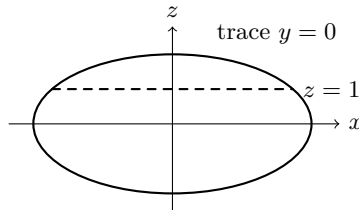
is cut by the plane $z = 1$.

Tasks

1. Find the section in standard form.
2. Determine its semiaxes and area.
3. Compare it quantitatively with the central section $z = 0$.

Question 7 – Solution

Strategy Substitute the fixed height and rewrite the remaining equation in standard ellipse form.



Step 1: Substitute With $z = 1$,

$$\frac{x^2}{16} + \frac{y^2}{9} + \frac{1}{4} = 1 \implies \frac{x^2}{16} + \frac{y^2}{9} = \frac{3}{4}.$$

Divide by $3/4$:

$$\boxed{\frac{x^2}{12} + \frac{y^2}{27/4} = 1}.$$

The semiaxes are $\boxed{2\sqrt{3}}$ and $\boxed{3\sqrt{3}/2}$.

Step 2: Area Therefore

$$A = \pi ab = \pi(2\sqrt{3})(3\sqrt{3}/2) = \boxed{9\pi}.$$

Comparison At $z = 0$, the semiaxes are 4 and 3, so the central area is 12π . The section at $z = 1$ has area $3/4$ as large, matching the right side $3/4$ before normalization.