

Question 5

A centered quadric has horizontal traces only for $|z| \geq 3$, where

$$x^2 + y^2 = 4 \left(\frac{z^2}{9} - 1 \right).$$

Tasks

1. Reconstruct and classify the quadric.
2. Find its vertices and axis.
3. Derive both principal vertical traces.
4. Explain the empty central region.

Question 5 – Solution

Strategy Rearrange the trace family and interpret both its signs and its existence condition.

Step 1: Reconstruct Move terms and divide by 4:

$$x^2 + y^2 = \frac{4}{9}z^2 - 4 \implies \boxed{\frac{z^2}{9} - \frac{x^2}{4} - \frac{y^2}{4} = 1}.$$

Step 2: Geometry One positive square and two negative squares equal 1, so this is a two-sheet hyperboloid with z -axis. Setting $x = y = 0$ gives vertices $\boxed{(0, 0, \pm 3)}$.

Step 3: Vertical traces Setting $y = 0$ and $x = 0$ gives

$$\boxed{\frac{z^2}{9} - \frac{x^2}{4} = 1}, \quad \boxed{\frac{z^2}{9} - \frac{y^2}{4} = 1}.$$

Step 4: Gap Since $x^2 + y^2 \geq 0$, the given trace requires $z^2/9 - 1 \geq 0$, hence $|z| \geq 3$. No surface points lie in the central slab $-3 < z < 3$.