

Question 4

Analyze the hyperbolic paraboloid

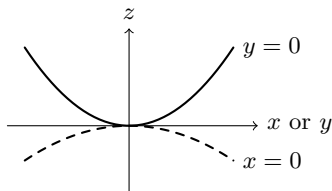
$$z = \frac{x^2}{4} - \frac{y^2}{9}.$$

Tasks

1. Derive the principal vertical traces.
2. Classify horizontal traces for $z = c$.
3. Find every line through the origin contained in the surface.
4. Justify the term “saddle.”

Question 4 – Solution

Strategy Fix one coordinate at a time, then factor the level-zero equation.



Step 1: Vertical traces If $y = 0$, then $z = x^2/4$, an upward parabola. If $x = 0$, then $z = -y^2/9$, a downward parabola. Their opposite concavities create the saddle.

Step 2: Horizontal traces At $z = c$,

$$\frac{x^2}{4} - \frac{y^2}{9} = c.$$

For $c > 0$ the hyperbola opens in the x -direction; for $c < 0$ it opens in the y -direction.

Step 3: Lines through the origin A contained line $(x, y, z) = t(a, b, c)$ must satisfy $tc = t^2(a^2/4 - b^2/9)$ for all t , so $c = 0$ and $a^2/4 = b^2/9$. Factoring gives

$$\left(\frac{x}{2} - \frac{y}{3}\right)\left(\frac{x}{2} + \frac{y}{3}\right) = 0.$$

Thus the two lines are $\boxed{z = 0, y = \pm 3x/2}$. Direct substitution confirms every point on each line lies on the surface.