

Question 3

For real k , classify

$$x^2 + y^2 - kz^2 = 1.$$

Tasks

1. Classify the surface for $k < 0$, $k = 0$, and $k > 0$.
2. Describe boundedness in each case.
3. Compare horizontal and vertical traces across the transition.

Question 3 – Solution

Strategy The sign of the z^2 coefficient controls the canonical type.

Case 1: $k < 0$ Write $k = -a$ with $a > 0$. Then $x^2 + y^2 + az^2 = 1$, an ellipsoid of revolution, bounded in every direction.

Case 2: $k = 0$ The equation becomes $x^2 + y^2 = 1$ with unrestricted z , so it is a circular cylinder of radius 1 parallel to the z -axis, unbounded in z .

Case 3: $k > 0$ The equation has two positive terms and one negative term, so it is a one-sheet hyperboloid about the z -axis. At $z = c$, the circle has radius $\sqrt{1 + kc^2}$; the surface is unbounded in all three coordinates.

Verification In every case the xy -trace is the unit circle. Vertical traces change from ellipses, to parallel lines, to hyperbolas, confirming the transition.