

Question 2

Analyze

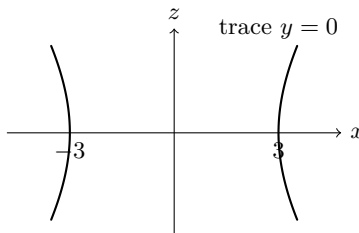
$$\frac{x^2}{9} + \frac{y^2}{4} - \frac{z^2}{16} = 1.$$

Tasks

1. Identify the surface and its axis.
2. Derive all three coordinate-plane traces.
3. Find the waist dimensions.
4. Explain why the surface is connected.

Question 2 – Solution

Strategy Read the signs and inspect traces one coordinate at a time.



Step 1: Classification Two positive squared terms and one negative term equal 1, so this is a hyperboloid of one sheet. The negative term is the z -term, hence the axis is the z -axis.

Step 2: Horizontal traces Setting $z = c$ gives

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 + \frac{c^2}{16}.$$

Every right side is positive, so every horizontal plane cuts an ellipse. At $c = 0$ the waist ellipse has semiaxes 3 and 2.

Step 3: Vertical traces At $y = 0$,

$$\frac{x^2}{9} - \frac{z^2}{16} = 1,$$

and at $x = 0$,

$$\frac{y^2}{4} - \frac{z^2}{16} = 1.$$

Both are hyperbolas opening perpendicular to the z -axis.

Conclusion Because a nonempty ellipse exists for every real $z = c$ and varies continuously with c , the surface is connected.