

Question 1

Classify and sketch the surface

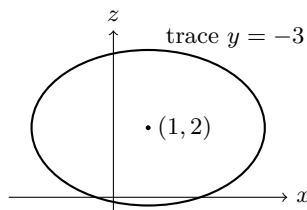
$$4x^2 + y^2 + 9z^2 - 8x + 6y - 36z + 4 = 0.$$

Tasks

1. Convert to standard form and classify it.
2. Find its center and semiaxes.
3. Derive its principal traces.
4. Find all coordinate-axis intercepts.

Question 1 – Solution

Strategy Group like variables and complete each square.



Step 1: Complete squares Write

$$4(x^2 - 2x) + (y^2 + 6y) + 9(z^2 - 4z) + 4 = 0.$$

Using $x^2 - 2x = (x - 1)^2 - 1$, $y^2 + 6y = (y + 3)^2 - 9$, and $z^2 - 4z = (z - 2)^2 - 4$ gives

$$4(x - 1)^2 + (y + 3)^2 + 9(z - 2)^2 = 45.$$

Divide by 45:

$$\frac{(x - 1)^2}{45/4} + \frac{(y + 3)^2}{45} + \frac{(z - 2)^2}{5} = 1.$$

Step 2: Geometry This is an ellipsoid centered at $(1, -3, 2)$ with semiaxes $3\sqrt{5}/2$, $3\sqrt{5}$, and $\sqrt{5}$. Its six axis endpoints relative to the center are obtained by adding and subtracting those lengths in the corresponding coordinate.

Step 3: Principal traces Through the center, the traces are

$$\begin{aligned} x = 1 : \frac{(y + 3)^2}{45} + \frac{(z - 2)^2}{5} &= 1, & y = -3 : \frac{(x - 1)^2}{45/4} + \frac{(z - 2)^2}{5} &= 1, \\ z = 2 : \frac{(x - 1)^2}{45/4} + \frac{(y + 3)^2}{45} &= 1. \end{aligned}$$

Step 4: Axis intercepts Set the other two coordinates to zero in the original equation. This gives

$$(1, 0, 0), \quad (0, -3 \pm \sqrt{5}, 0), \quad (0, 0, 2 \pm 4\sqrt{2}/3).$$

Substitution verifies each intercept and trace.