

Question 9

Find the locus of points equidistant from the intersecting planes

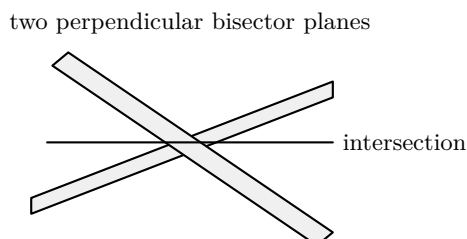
$$\Pi_1 : x - 2y + 2z - 3 = 0, \quad \Pi_2 : 2x + 2y - z + 6 = 0.$$

Tasks

1. Derive equations for both components of the locus.
2. Explain why they are angle-bisector planes.
3. Prove that the two resulting planes are perpendicular.

Question 9 – Solution

Strategy Use the point-to-plane distance formula with absolute values. Equal normal lengths simplify the two sign cases.



Step 1: Distance equation The normal lengths are

$$\sqrt{1^2 + (-2)^2 + 2^2} = 3, \quad \sqrt{2^2 + 2^2 + (-1)^2} = 3.$$

Thus equality of point-to-plane distances becomes

$$|x - 2y + 2z - 3| = |2x + 2y - z + 6|.$$

Step 2: Resolve the absolute values Let the two expressions be F_1 and F_2 . The equation $|F_1| = |F_2|$ gives two cases.

If $F_1 = F_2$, then

$$x - 2y + 2z - 3 = 2x + 2y - z + 6 \implies x + 4y - 3z + 9 = 0.$$

If $F_1 = -F_2$, then

$$x - 2y + 2z - 3 = -2x - 2y + z - 6 \implies 3x + z + 3 = 0.$$

Therefore the locus consists of

$$\boxed{x + 4y - 3z + 9 = 0}, \quad \boxed{3x + z + 3 = 0}.$$

Interpretation On either plane, the signed perpendicular displacements from Π_1 and Π_2 have equal magnitude. These are precisely the internal and external angle-bisector planes, and both contain the original planes' intersection line.

Perpendicularity Normals may be chosen as $\langle 1, 4, -3 \rangle$ and $\langle 3, 0, 1 \rangle$. Their dot product is $3 - 3 = 0$, so the bisector planes are perpendicular.

Verification Substituting any point from either boxed plane into the absolute-value equation recovers equal distances.