

Question 5

Let $P = (4, -1, 5)$ and

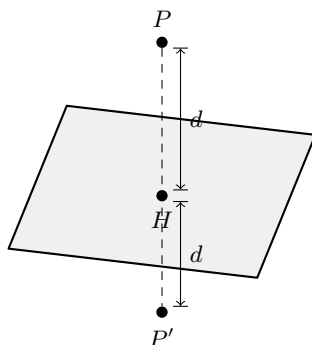
$$\Pi : 2x - y + 2z = 3.$$

Tasks

1. Find the perpendicular projection H of P onto Π .
2. Find the distance from P to Π .
3. Reflect P across Π and find the image point P' .
4. Verify the midpoint and perpendicularity conditions.

Question 5 – Solution

Strategy Move from P along the normal $\vec{n} = \langle 2, -1, 2 \rangle$. The signed equation value determines the required multiple.



Step 1: Signed displacement Write the plane as $F(x, y, z) = 2x - y + 2z - 3 = 0$. At P ,

$$2(4) - (-1) + 2(5) - 3 = 16, \quad \|\vec{n}\|^2 = 9.$$

Step 2: Projection A point on the perpendicular through P is $P - s\vec{n}$. To reach the plane,

$$F(P - s\vec{n}) = F(P) - s\|\vec{n}\|^2 = 16 - 9s = 0,$$

so $s = 16/9$. Hence

$$H = P - \frac{16}{9}\vec{n} = \left(4 - \frac{32}{9}, -1 + \frac{16}{9}, 5 - \frac{32}{9}\right) = \boxed{\left(\frac{4}{9}, \frac{7}{9}, \frac{13}{9}\right)}.$$

Step 3: Distance Since $P - H = (16/9)\vec{n}$,

$$d(P, \Pi) = \frac{16}{9}\|\vec{n}\| = \frac{16}{9}(3), \quad \boxed{d(P, \Pi) = \frac{16}{3}}.$$

Step 4: Reflection Since H is the midpoint of PP' ,

$$P' = 2H - P = P - \frac{32}{9}\vec{n} = \boxed{\left(-\frac{28}{9}, \frac{23}{9}, -\frac{19}{9}\right)}.$$

Verification Substituting H into the plane gives 3. Also $P - H = (16/9)\vec{n}$, so PH is perpendicular to the plane, and direct averaging of P and P' gives H .