

Question 4

Consider the planes

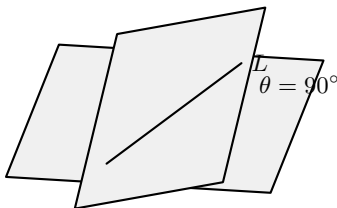
$$\Pi_1 : x + y - z = 2, \quad \Pi_2 : 2x - y + z = 1.$$

Tasks

1. Find a vector and parametric equation for their line of intersection.
2. Find the dihedral angle between the planes.
3. Verify the line equation in both plane equations.

Question 4 – Solution

Strategy The intersection direction is perpendicular to both normals. Solve the two plane equations for one point and one free parameter.



Step 1: Solve simultaneously Add the equations:

$$(x + y - z) + (2x - y + z) = 2 + 1 \implies 3x = 3 \implies x = 1.$$

Substitute $x = 1$ into $x + y - z = 2$:

$$1 + y - z = 2 \implies y = 1 + z.$$

Let the free coordinate be $z = t$. Then $y = 1 + t$, so

$$\boxed{\vec{r} = \langle 1, 1, 0 \rangle + t \langle 0, 1, 1 \rangle}.$$

Step 2: Confirm the direction The normals are $\vec{n}_1 = \langle 1, 1, -1 \rangle$ and $\vec{n}_2 = \langle 2, -1, 1 \rangle$. Their cross product is $\langle 0, -3, -3 \rangle$, parallel to $\langle 0, 1, 1 \rangle$.

Step 3: Angle Compute $\vec{n}_1 \cdot \vec{n}_2 = 2 - 1 - 1 = 0$. Therefore

$$\cos \theta = \frac{|\vec{n}_1 \cdot \vec{n}_2|}{\|\vec{n}_1\| \|\vec{n}_2\|} = 0, \quad \boxed{\theta = 90^\circ}.$$

Verification Substitution of $(1, 1 + t, t)$ gives 2 in Π_1 and 1 in Π_2 for every t .