

Question 2

A plane contains the line

$$L : \vec{r} = \langle 1, -2, 0 \rangle + t \langle 2, 1, -1 \rangle$$

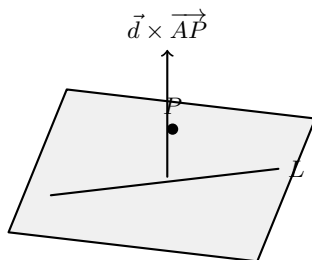
and the point $P = (0, 1, 4)$.

Tasks

1. Find a Cartesian equation of the plane.
2. Explain why the data determine a unique plane.
3. Verify that the entire line and the point lie in the plane.

Question 2 – Solution

Strategy Combine the line direction with a vector from a point on the line to P . Their cross product is a plane normal.



Step 1: Two directions in the plane Take $A = (1, -2, 0)$ from the line. Its direction is $\vec{d} = \langle 2, 1, -1 \rangle$, while

$$\overrightarrow{AP} = P - A = \langle 0 - 1, 1 - (-2), 4 - 0 \rangle = \langle -1, 3, 4 \rangle.$$

Step 2: Find a normal Compute

$$\vec{d} \times \overrightarrow{AP} = \langle 1(4) - (-1)(3), (-1)(-1) - 2(4), 2(3) - 1(-1) \rangle = \langle 7, -7, 7 \rangle.$$

Thus $\vec{n} = \langle 1, -1, 1 \rangle$. Using point A ,

$$(x - 1) - (y + 2) + (z - 0) = 0,$$

so the plane is

$$\boxed{x - y + z = 3}.$$

Uniqueness The vectors $\vec{d}$ and $\overrightarrow{AP}$ are not parallel, so they span exactly one plane.

Verification Point P gives $0 - 1 + 4 = 3$. A general point on L is $(1 + 2t, -2 + t, -t)$, and

$$(1 + 2t) - (-2 + t) - t = 3$$

for every t , proving that the whole line is contained in the plane.