

Question 1

A plane passes through

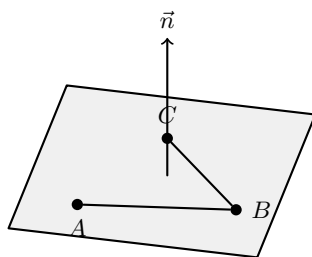
$$A = (1, 0, 2), \quad B = (3, 1, -1), \quad C = (-1, 4, 0).$$

Tasks

1. Derive a Cartesian equation of the plane from two in-plane vectors.
2. Write a unit normal vector.
3. Find all three coordinate-axis intercepts and verify the three given points.

Question 1 – Solution

Strategy Form $\overrightarrow{AB}$ and $\overrightarrow{AC}$; their cross product is normal to the plane.



Step 1: In-plane vectors Subtract coordinates component by component:

$$\begin{aligned}\overrightarrow{AB} &= B - A = \langle 3 - 1, 1 - 0, -1 - 2 \rangle = \langle 2, 1, -3 \rangle, \\ \overrightarrow{AC} &= C - A = \langle -1 - 1, 4 - 0, 0 - 2 \rangle = \langle -2, 4, -2 \rangle.\end{aligned}$$

Step 2: Normal vector Their cross product is

$$\overrightarrow{AB} \times \overrightarrow{AC} = \langle 1(-2) - (-3)(4), (-3)(-2) - 2(-2), 2(4) - 1(-2) \rangle = \langle 10, 10, 10 \rangle.$$

Divide by 10 to use the primitive normal $\vec{n} = \langle 1, 1, 1 \rangle$. The point-normal equation through A is

$$(x - 1) + (y - 0) + (z - 2) = 0.$$

Therefore

$$\boxed{x + y + z = 3}, \quad \boxed{\hat{n} = \frac{1}{\sqrt{3}} \langle 1, 1, 1 \rangle}.$$

Step 3: Intercepts On the x -axis, $y = z = 0$, so $x = 3$. The other axes work similarly:

$$\boxed{(3, 0, 0), (0, 3, 0), (0, 0, 3)}.$$

Verification Substitution of A , B , and C gives 3 each time. The two in-plane vectors dot to zero with $\langle 1, 1, 1 \rangle$, independently confirming the orientation.