

Question 10

Two particles move for $t \geq 0$ according to

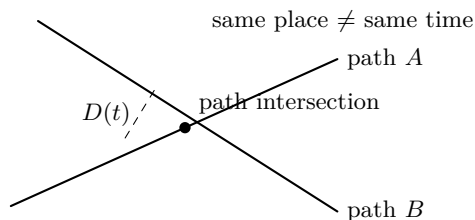
$$\begin{aligned}\vec{r}_A(t) &= \langle 0, 1, 2 \rangle + t \langle 2, -1, 1 \rangle, \\ \vec{r}_B(t) &= \langle 6, -2, 5 \rangle + t \langle -1, 1, 0 \rangle.\end{aligned}$$

Tasks

1. Determine whether their geometric paths intersect.
2. Determine whether the particles collide.
3. Find the time and value of their minimum separation for $t \geq 0$.

Question 10 – Solution

Strategy Path intersection allows different parameters, whereas collision uses the same time. Minimize the squared norm of the relative-position vector.



Paths versus collision Solving $\vec{r}_A(s) = \vec{r}_B(u)$ gives $s = 3$, $u = 0$, so the paths intersect at $\boxed{(6, -2, 5)}$. The particles reach that point at different times and therefore do not collide. Equating positions at the same t would require all components of the relative vector below to vanish, which never occurs.

Separation At a common time t ,

$$\vec{r}_A(t) - \vec{r}_B(t) = \langle -6 + 3t, 3 - 2t, -3 + t \rangle.$$

Hence

$$D^2(t) = (-6 + 3t)^2 + (3 - 2t)^2 + (-3 + t)^2 = 14t^2 - 54t + 54.$$

The vertex occurs at

$$t = \frac{54}{28} = \frac{27}{14} > 0.$$

Substitution gives $D_{\min}^2 = 27/14$, so

$$\boxed{D_{\min} = \sqrt{\frac{27}{14}} = \frac{3\sqrt{42}}{14}}.$$

Verification The quadratic opens upward, and the minimizing time lies in the allowed domain. The nonzero minimum confirms independently that no collision occurs.