

Question 9

A line intersects the xy -plane at

$$A = (2, -1, 0)$$

and the yz -plane at

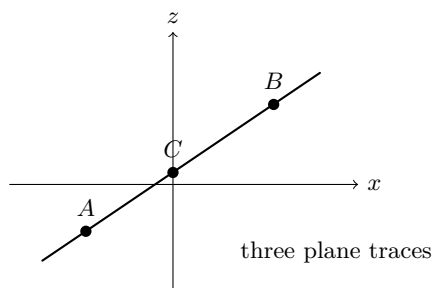
$$B = (0, 3, 4).$$

Tasks

1. Write vector, parametric, and symmetric equations of the line.
2. Find its intersection with the xz -plane.
3. Determine the ratio in which the xz -plane intersection divides $\overline{AB}$.

Question 9 – Solution

Strategy The two traces determine the line. Parameterize from A toward B , then impose $y = 0$ for the xz -plane.



Line equations A direction is $B - A = \langle -2, 4, 4 \rangle = 2 \langle -1, 2, 2 \rangle$. Thus

$$\vec{r} = \langle 2, -1, 0 \rangle + t \langle -1, 2, 2 \rangle,$$

$$x = 2 - t, \quad y = -1 + 2t, \quad z = 2t,$$

and

$$\frac{x - 2}{-1} = \frac{y + 1}{2} = \frac{z}{2}.$$

xz -plane trace Set $y = 0$. Then $-1 + 2t = 0$, so $t = 1/2$ and

$$C = \left(\frac{3}{2}, 0, 1 \right).$$

Division ratio In this reduced parameterization, B occurs at $t = 2$. Thus C lies one-half unit from A in parameter and three-halves units from B , giving

$$AC : CB = 1 : 3.$$

Verification C has $y = 0$, and substituting its coordinates into the symmetric equation gives the common value $1/2$.