

Question 8

Two lines intersect at $Q = (1, -2, 3)$ and have direction vectors

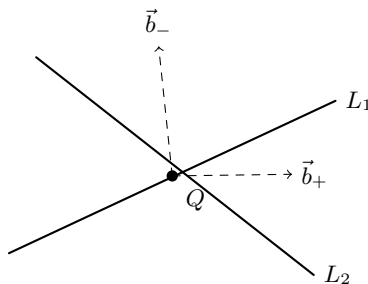
$$\vec{d}_1 = \langle 3, 0, 4 \rangle, \quad \vec{d}_2 = \langle 0, 4, 3 \rangle.$$

Tasks

1. Find equations of both angle-bisector lines through Q .
2. Determine which bisector corresponds to the acute angle between the given lines.
3. Prove that the two bisector lines are perpendicular.

Question 8 – Solution

Strategy Angle-bisector directions are the sum and difference of unit direction vectors. Here the given vectors already have equal length.



Bisector directions Since $\|\vec{d}_1\| = \|\vec{d}_2\| = 5$, the sum and difference give

$$\vec{b}_+ = \langle 3, 4, 7 \rangle, \quad \vec{b}_- = \langle 3, -4, 1 \rangle.$$

Therefore the bisectors are

$$\boxed{\vec{r} = \langle 1, -2, 3 \rangle + t \langle 3, 4, 7 \rangle}, \quad \boxed{\vec{r} = \langle 1, -2, 3 \rangle + u \langle 3, -4, 1 \rangle}.$$

Acute-angle bisector Because $\vec{d}_1 \cdot \vec{d}_2 = 12 > 0$, the directions form an acute angle. Their sum $\vec{b}_+$ lies between them and bisects that acute angle; $\vec{b}_-$ bisects the supplementary angle.

Perpendicularity Directly,

$$\vec{b}_+ \cdot \vec{b}_- = 3(3) + 4(-4) + 7(1) = 9 - 16 + 7 = 0.$$

Hence the two angle-bisector lines are perpendicular.

Verification Dotting $\vec{b}_+$ with each original direction gives the same normalized cosine, confirming equal angles.