

Question 7

A line through

$$P = (-1, 2, 0)$$

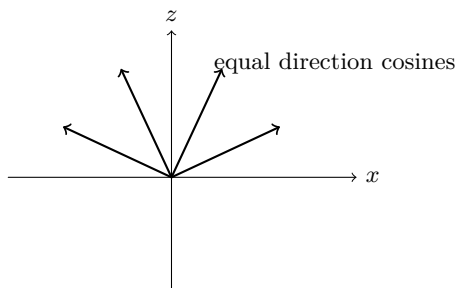
makes equal acute angles with the x -, y -, and z -axes (treated as unoriented lines). Here the first octant includes its boundary planes.

Tasks

1. Find all geometrically distinct lines through P having this property.
2. Determine which of these lines contain at least one point in the first octant.
3. State the parameter intervals corresponding to the portions lying in the first octant.

Question 7 – Solution

Strategy Equal inclination means the three direction cosines have equal absolute value. Direction vectors that differ only by an overall nonzero factor define the same line.



All lines Up to reversal, there are four sign patterns. The distinct lines are

$$\vec{r} = \langle -1, 2, 0 \rangle + t\vec{d}$$

where

$$\vec{d} \in \{ \langle 1, 1, 1 \rangle, \langle 1, 1, -1 \rangle, \langle 1, -1, 1 \rangle, \langle 1, -1, -1 \rangle \}.$$

Each direction has direction cosines with absolute value $1/\sqrt{3}$.

First-octant test For $\langle 1, 1, 1 \rangle$, the coordinates are $(-1 + t, 2 + t, t)$, all nonnegative when $t \geq 1$. For $\langle 1, -1, 1 \rangle$, they are $(-1 + t, 2 - t, t)$, all nonnegative when $1 \leq t \leq 2$.

For $\langle 1, 1, -1 \rangle$ and $\langle 1, -1, -1 \rangle$, the requirements $x \geq 0$ and $z \geq 0$ force incompatible signs of t . Thus exactly the first and third listed lines enter the first octant.

Verification The stated intervals satisfy all three coordinate inequalities, including the boundary planes.