

Question 6

Consider the two lines

$$L_1 : \vec{r} = s \langle 1, 2, 0 \rangle ,$$

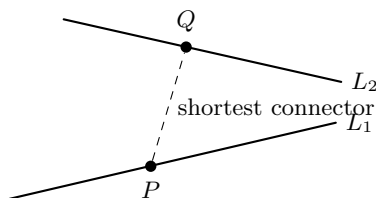
$$L_2 : \vec{r} = \langle 1, 0, 3 \rangle + t \langle 2, -1, 1 \rangle .$$

Tasks

1. Show that the lines are skew.
2. Find the point on each line at which their shortest connecting segment begins and ends.
3. Determine the distance between the lines.

Question 6 – Solution

Strategy For closest points $P \in L_1$ and $Q \in L_2$, the connector $Q - P$ must be perpendicular to both line directions.



Classification The directions $\langle 1, 2, 0 \rangle$ and $\langle 2, -1, 1 \rangle$ are not parallel. Solving the first two intersection equations gives $s = 1/5$, $t = -2/5$, but the third coordinates are then 0 and $13/5$, so the lines do not intersect. They are skew.

Closest points Let

$$P = (s, 2s, 0), \quad Q = (1 + 2t, -t, 3 + t).$$

The conditions $(Q - P) \cdot \langle 1, 2, 0 \rangle = 0$ and $(Q - P) \cdot \langle 2, -1, 1 \rangle = 0$ yield

$$s = \frac{1}{5}, \quad t = -\frac{5}{6}.$$

Thus

$$\boxed{P = \left(\frac{1}{5}, \frac{2}{5}, 0\right)}, \quad \boxed{Q = \left(-\frac{2}{3}, \frac{5}{6}, \frac{13}{6}\right)}.$$

Distance and verification Since $Q - P = \frac{13}{30} \langle -2, 1, 5 \rangle$,

$$\boxed{d(L_1, L_2) = \frac{13\sqrt{30}}{30} = \frac{13}{\sqrt{30}}}.$$

The vector $\langle -2, 1, 5 \rangle$ has zero dot product with both line directions.