

**Question 5**

Let

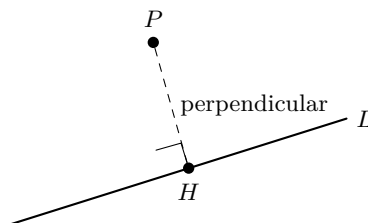
$$P = (4, 3, -1), \quad L : \vec{r} = \langle 1, -1, 2 \rangle + t \langle 2, 1, 2 \rangle.$$

**Tasks**

1. Find the point  $H$  on  $L$  closest to  $P$ .
2. Write an equation of the line through  $P$  that intersects  $L$  perpendicularly at  $H$ .
3. Find the distance from  $P$  to  $L$ .

## Question 5 – Solution

**Strategy** At the closest point, the displacement from the line to  $P$  is perpendicular to the line's direction vector.



**Perpendicular foot** Write  $A = (1, -1, 2)$  and  $\vec{d} = \langle 2, 1, 2 \rangle$ . For  $H = A + t\vec{d}$ ,

$$(P - H) \cdot \vec{d} = 0 \implies (P - A - t\vec{d}) \cdot \vec{d} = 0.$$

Since  $(P - A) \cdot \vec{d} = 4$  and  $\vec{d} \cdot \vec{d} = 9$ ,  $t = 4/9$ . Therefore

$$H = \left( \frac{17}{9}, -\frac{5}{9}, \frac{26}{9} \right).$$

**Perpendicular line and distance** A direction from  $P$  to  $H$  is  $\langle -19, -32, 35 \rangle$  after multiplying by 9. Thus

$$\vec{r} = \langle 4, 3, -1 \rangle + u \langle -19, -32, 35 \rangle.$$

Also,

$$d(P, L) = \|P - H\| = \frac{\sqrt{2610}}{9} = \frac{\sqrt{290}}{3}.$$

**Verification**  $\langle -19, -32, 35 \rangle \cdot \langle 2, 1, 2 \rangle = -38 - 32 + 70 = 0$ , and  $H$  satisfies the original line equation.