

Question 4

For a real parameter k , consider

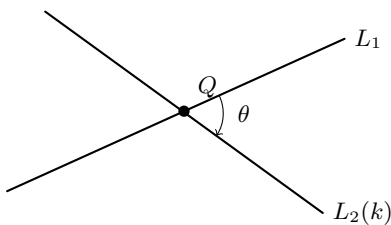
$$\begin{aligned}L_1 : \vec{r} &= \langle 1, 2, 0 \rangle + s \langle 2, -1, 3 \rangle, \\L_2(k) : \vec{r} &= \langle 4, -2, 7 \rangle + t \langle 1, k, -1 \rangle.\end{aligned}$$

Tasks

1. Find the value of k for which the lines intersect.
2. Find the intersection point.
3. For that value of k , find the acute angle between the lines.

Question 4 – Solution

Strategy The x - and z -coordinate equations determine s and t without k ; the y -coordinate then determines the required parameter.



Intersection Equating x - and z -coordinates gives

$$1 + 2s = 4 + t, \quad 3s = 7 - t.$$

The unique solution is $s = 2$, $t = 1$. The y -coordinates then require

$$2 - s = -2 + kt \implies 0 = -2 + k,$$

so $k = 2$. Both lines meet at $Q = (5, 0, 6)$.

Angle The directions are $\vec{d}_1 = \langle 2, -1, 3 \rangle$ and $\vec{d}_2 = \langle 1, 2, -1 \rangle$. Thus

$$\cos \theta = \frac{|\vec{d}_1 \cdot \vec{d}_2|}{\|\vec{d}_1\| \|\vec{d}_2\|} = \frac{3}{\sqrt{14}\sqrt{6}} = \frac{3}{2\sqrt{21}}.$$

Hence $\theta = \cos^{-1}\left(\frac{3}{2\sqrt{21}}\right) \approx 70.9^\circ$.

Verification Substitution of $s = 2$, $t = 1$, and $k = 2$ produces $(5, 0, 6)$ in both equations.