

Question 3

Points

$$A = (1, -2, 3), \quad B = (5, 4, -1), \quad P = (4, u, v)$$

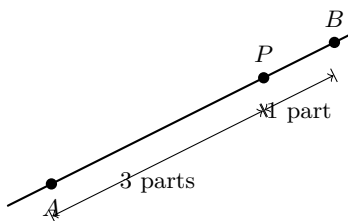
are collinear, and P lies between A and B .

Tasks

1. Determine u and v .
2. Write a parametric equation of the common line.
3. Find the ratio $AP : PB$ and verify it using distances.

Question 3 – Solution

Strategy Parameterize the segment from A to B . The known x -coordinate of P determines the parameter, after which the other coordinates and division ratio follow.



Parameter and coordinates Since $B - A = \langle 4, 6, -4 \rangle$,

$$\vec{r}(t) = \langle 1, -2, 3 \rangle + t \langle 4, 6, -4 \rangle.$$

At P , $1 + 4t = 4$, so $t = 3/4$. Therefore

$$u = -2 + 6\left(\frac{3}{4}\right) = \frac{5}{2}, \quad v = 3 - 4\left(\frac{3}{4}\right) = 0,$$

and

$$\boxed{P = \left(4, \frac{5}{2}, 0\right)}.$$

Division ratio The parameter places P three-fourths of the way from A to B , so

$$\boxed{AP : PB = 3 : 1}.$$

Verification We have $P - A = \frac{3}{4}(B - A)$ and $B - P = \frac{1}{4}(B - A)$; taking lengths preserves the ratio $3 : 1$.