

Question 2

Consider the line

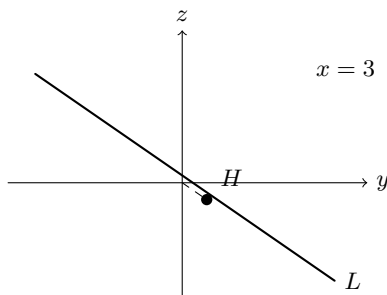
$$L : \quad x = 3, \quad \frac{y+2}{4} = \frac{z-1}{-3}.$$

Tasks

1. Write vector and parametric equations for L .
2. Explain geometrically what the equation $x = 3$ says about the line.
3. Find the point on L closest to the origin and determine that minimum distance.

Question 2 – Solution

Strategy Use the common symmetric ratio as a parameter. For the closest point, minimize the squared distance to avoid unnecessary square roots.



Representations Let the common ratio equal t . Then

$$\vec{r} = \langle 3, -2, 1 \rangle + t \langle 0, 4, -3 \rangle, \quad x = 3, \quad y = -2 + 4t, \quad z = 1 - 3t.$$

The zero x -component of the direction means the entire line lies in the plane $x = 3$ and is parallel to the yz -plane.

Closest point The squared distance from the origin is

$$D^2(t) = 9 + (-2 + 4t)^2 + (1 - 3t)^2 = 25t^2 - 22t + 14.$$

Its minimum occurs at $t = 22/50 = 11/25$. Hence

$$H = \left(3, -\frac{6}{25}, -\frac{8}{25} \right), \quad d(O, L) = \frac{\sqrt{229}}{5}.$$

Verification The vector H from the origin has dot product 0 with $\langle 0, 4, -3 \rangle$, confirming that OH is perpendicular to L .