

Question 1

A line passes through the points

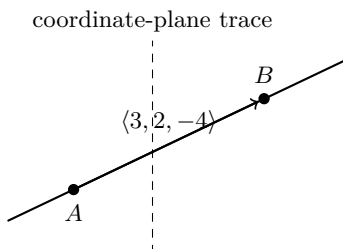
$$A = (-2, 1, 3), \quad B = (4, 5, -5).$$

Tasks

1. Write the line in vector, parametric, and symmetric forms using a primitive integer direction vector.
2. Find the line's intersection with each coordinate plane.
3. Verify that the three plane-intersection points lie on your line.

Question 1 – Solution

Strategy Subtract the given points to obtain a direction, reduce it, and set one coordinate at a time equal to zero to find the coordinate-plane traces.



Line equations Since $B - A = \langle 6, 4, -8 \rangle = 2 \langle 3, 2, -4 \rangle$,

$$\vec{r} = \langle -2, 1, 3 \rangle + t \langle 3, 2, -4 \rangle,$$

$$x = -2 + 3t, \quad y = 1 + 2t, \quad z = 3 - 4t,$$

$$\frac{x + 2}{3} = \frac{y - 1}{2} = \frac{z - 3}{-4}.$$

Coordinate-plane traces Setting $z = 0$, $x = 0$, and $y = 0$ gives, respectively,

$$L \cap xy = \left(\frac{1}{4}, \frac{5}{2}, 0 \right), \quad L \cap yz = \left(0, \frac{7}{3}, \frac{1}{3} \right), \quad L \cap xz = \left(-\frac{7}{2}, 0, 5 \right).$$

Verification These correspond to $t = 3/4$, $2/3$, and $-1/2$ in the same parametric equations, so all three points lie on the line.