

Question 9

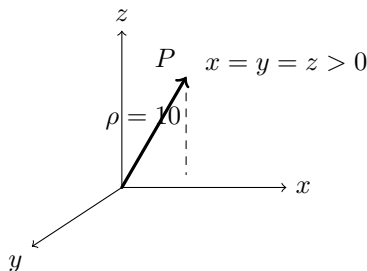
A point P has spherical coordinates (ρ, θ, ϕ) with $\rho = 10$ and is equally distant from the positive x -, y -, and z -axes. Assume P lies in the first octant.

Tasks

1. Determine the Cartesian coordinates of P .
2. Find its principal spherical angles.
3. Verify the three axis distances directly.

Question 9 – Solution

Strategy. Express squared distance to each coordinate axis and use first-octant signs.



Step 1: Equal-distance conditions Squared distances to the x -, y -, and z -axes are $y^2 + z^2$, $x^2 + z^2$, and $x^2 + y^2$. Equality implies $x^2 = y^2 = z^2$. In the first octant, $x = y = z$.

Step 2: Radius condition Since $x^2 + y^2 + z^2 = 100$, we obtain $3x^2 = 100$ and

$$P = (10/\sqrt{3}, 10/\sqrt{3}, 10/\sqrt{3}).$$

Then $\theta = \pi/4$, and $\cos \phi = z/\rho = 1/\sqrt{3}$, so

$$(\rho, \theta, \phi) = (10, \pi/4, \arccos(1/\sqrt{3})).$$

Verification. Every squared axis distance is $100/3 + 100/3 = 200/3$, so every distance is $10\sqrt{2/3}$.