

Question 8

Analyze all spherical-coordinate representations of points on the z -axis under the conventions $\rho \geq 0$, $0 \leq \theta < 2\pi$, and $0 \leq \phi \leq \pi$.

Tasks

1. Characterize a point $(0, 0, a)$ for $a > 0$, $a < 0$, and $a = 0$.
2. Identify which coordinates are nonunique.
3. Explain why the conversion formulas remain consistent.

Question 8 – Solution

Strategy. On the axis, the horizontal projection has length $\rho \sin \phi = 0$, so analyze the poles and origin separately.

Step 1: Positive axis If $a > 0$, then $\rho = a$, $\phi = 0$, and θ is arbitrary:

$$\boxed{(a, \theta, 0), \quad 0 \leq \theta < 2\pi}.$$

Step 2: Negative axis If $a < 0$, then $\rho = |a|$, $\phi = \pi$, and again θ is arbitrary:

$$\boxed{(|a|, \theta, \pi), \quad 0 \leq \theta < 2\pi}.$$

At the origin, $\rho = 0$ and both angles are arbitrary in their stated ranges.

Step 3: Consistency At $\phi = 0$ or π , $\sin \phi = 0$, so $x = y = 0$ regardless of θ , while $z = \rho \cos \phi$ has the correct sign. At $\rho = 0$, all three Cartesian coordinates vanish regardless of either angle. Thus the nonuniqueness is coordinate degeneracy, not geometric ambiguity.