

Question 6

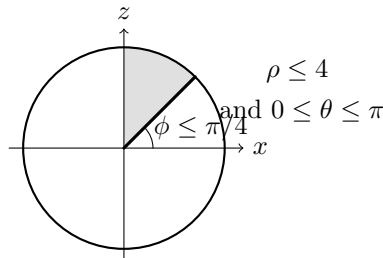
Describe in spherical inequalities the solid inside $x^2 + y^2 + z^2 \leq 16$, above the cone $z = \sqrt{x^2 + y^2}$, and in the half-space $y \geq 0$.

Tasks

1. Determine bounds for ρ , θ , and ϕ .
2. Explain each angular bound geometrically.
3. State which boundary pieces meet at the origin.

Question 6 – Solution

Strategy. Translate the sphere to a radial bound, the half-space to azimuth, and compare z with the cylindrical radius to bound inclination.



Step 1: Radial bound The sphere gives $0 \leq \rho \leq 4$.

Step 2: Azimuth Since $y = \rho \sin \phi \sin \theta$ and $\sin \phi \geq 0$, $y \geq 0$ is represented without overlap by $0 \leq \theta \leq \pi$.

Step 3: Inclination Above the cone means

$$\rho \cos \phi \geq \rho \sin \phi.$$

For $\rho > 0$ this gives $0 \leq \phi \leq \pi/4$. Consequently

$$\boxed{0 \leq \rho \leq 4, \quad 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq \pi/4}.$$

The sphere is the outer boundary; the cone and the two half-plane faces $\theta = 0, \pi$ all meet at the origin. Angular multiplicity there does not duplicate physical volume.