

Question 5

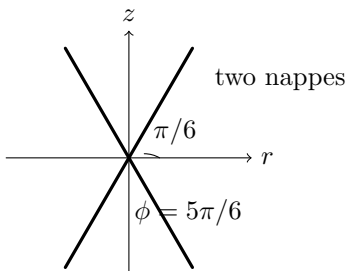
Convert the surface $z^2 = 3(x^2 + y^2)$ to spherical coordinates and determine exactly which cone it represents.

Tasks

1. Derive all permitted constant values of ϕ .
2. Identify both nappes.
3. State the status of the origin.

Question 5 – Solution

Strategy. Substitute $z = \rho \cos \phi$ and $x^2 + y^2 = \rho^2 \sin^2 \phi$, treating $\rho = 0$ separately.



Step 1: Convert For $\rho > 0$,

$$\rho^2 \cos^2 \phi = 3\rho^2 \sin^2 \phi \implies \tan^2 \phi = \frac{1}{3}.$$

On $0 \leq \phi \leq \pi$, the solutions are

$$\boxed{\phi = \frac{\pi}{6} \quad \text{or} \quad \phi = \frac{5\pi}{6}}.$$

Step 2: Geometry The first value is the upper nappe, $z \geq 0$; the second is the lower nappe, $z \leq 0$. Each makes an acute angle $\pi/6$ with its adjacent half of the z -axis.

Step 3: Origin At $\rho = 0$, every angle represents the origin, which satisfies the Cartesian equation. Therefore the two nappes meet at the included vertex $\boxed{(0, 0, 0)}$.