

Question 4

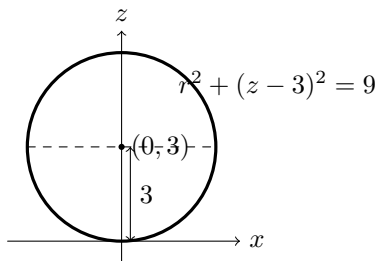
Rewrite the sphere $x^2 + y^2 + z^2 = 6z$ in spherical coordinates and identify its geometry.

Tasks

1. Obtain the nontrivial spherical equation.
2. Find its Cartesian center and radius.
3. Explain the role of $\rho = 0$ when simplifying.

Question 4 – Solution

Strategy. Substitute $x^2 + y^2 + z^2 = \rho^2$ and $z = \rho \cos \phi$, while preserving any solution lost by division.

**Step 1: Convert**

$$\rho^2 = 6\rho \cos \phi \iff \rho(\rho - 6 \cos \phi) = 0.$$

The usual surface equation is

$$\boxed{\rho = 6 \cos \phi}, \quad 0 \leq \phi \leq \frac{\pi}{2},$$

and it includes the origin at $\phi = \pi/2$.

Step 2: Identify Completing the square in Cartesian form gives

$$x^2 + y^2 + (z - 3)^2 = 9.$$

It is the sphere centered at $\boxed{(0, 0, 3)}$ with radius $\boxed{3}$.

Step 3: Origin check Division by ρ would temporarily discard $\rho = 0$. The origin satisfies the original equation and is recovered by $\rho = 6 \cos(\pi/2) = 0$; thus no point is absent from the final parametrized surface.