

Question 3

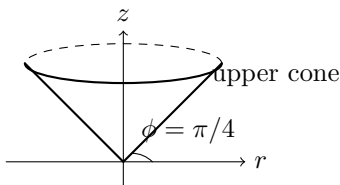
Describe geometrically the coordinate surfaces $\rho = 5$, $\theta = 2\pi/3$, and $\phi = \pi/4$.

Tasks

1. Identify each full coordinate surface.
2. Describe the pairwise intersections.
3. Find their common point.

Question 3 – Solution

Strategy. Interpret ρ as distance, θ as azimuth, and ϕ as inclination from the positive z -axis.



Step 1: Surfaces $\rho = 5$ is the sphere of radius 5. The equation $\theta = 2\pi/3$ is a vertical half-plane from the z -axis in azimuth $2\pi/3$. The equation $\phi = \pi/4$ is the upper nappe of the cone making angle $\pi/4$ with the positive z -axis.

Step 2: Pairwise intersections Sphere–half-plane and sphere–cone intersections are semicircle and horizontal circle, respectively. Half-plane–cone is a ray from the origin.

Step 3: Common point Substitution gives

$$x = 5\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) = -\frac{5\sqrt{2}}{4}, \quad y = 5\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{5\sqrt{6}}{4}, \quad z = \frac{5\sqrt{2}}{2}.$$

Thus

$$P = \left(-5\sqrt{2}/4, 5\sqrt{6}/4, 5\sqrt{2}/2\right).$$