

Question 2

Convert $(\rho, \theta, \phi) = (8, 5\pi/6, \pi/3)$ to Cartesian and cylindrical coordinates.

Tasks

1. Find (x, y, z) exactly.
2. Find the corresponding principal cylindrical coordinates.
3. Verify both radial identities.

Question 2 – Solution

Strategy. Use $r = \rho \sin \phi$ and $z = \rho \cos \phi$, then resolve r using θ .

Step 1: Cylindrical data

$$r = 8 \sin(\pi/3) = 4\sqrt{3}, \quad z = 8 \cos(\pi/3) = 4.$$

Thus the principal cylindrical coordinates are

$$(r, \theta, z) = (4\sqrt{3}, 5\pi/6, 4).$$

Step 2: Cartesian data

$$x = r \cos \theta = (4\sqrt{3})(-\sqrt{3}/2) = -6, \quad y = r \sin \theta = (4\sqrt{3})(1/2) = 2\sqrt{3}.$$

Therefore

$$(x, y, z) = (-6, 2\sqrt{3}, 4).$$

Verification. $x^2 + y^2 = 36 + 12 = 48 = r^2$, and $x^2 + y^2 + z^2 = 48 + 16 = 64 = \rho^2$.