

Question 1

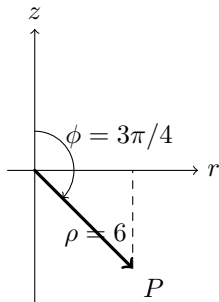
Convert the Cartesian point $P = (-3, 3, -3\sqrt{2})$ to spherical coordinates using $\rho \geq 0$, $0 \leq \theta < 2\pi$, and $0 \leq \phi \leq \pi$.

Tasks

1. Find ρ , θ , and ϕ .
2. Give one equivalent spherical representation outside the principal angular range.
3. Verify by converting back.

Question 1 – Solution

Strategy. Find radial distance first, use the quadrant of (x, y) for θ , and use $\cos \phi = z/\rho$.



Step 1: Radius

$$\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{9 + 9 + 18} = 6.$$

Step 2: Angles The projection $(-3, 3)$ lies in quadrant II, so $\theta = 3\pi/4$. Also

$$\cos \phi = \frac{-3\sqrt{2}}{6} = -\frac{\sqrt{2}}{2}, \quad 0 \leq \phi \leq \pi,$$

giving $\phi = 3\pi/4$. Hence

$$\boxed{(\rho, \theta, \phi) = (6, 3\pi/4, 3\pi/4)}.$$

Step 3: Equivalence and verification Adding 2π to the azimuth gives $(6, 11\pi/4, 3\pi/4)$. Moreover,

$$6 \sin(3\pi/4) \cos(3\pi/4) = -3, \quad 6 \sin(3\pi/4) \sin(3\pi/4) = 3, \quad 6 \cos(3\pi/4) = -3\sqrt{2}.$$