

Question 10

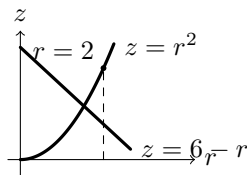
A solid is described cylindrically by $0 \leq r \leq 2 + \cos \theta$, $0 \leq \theta < 2\pi$, and $r^2 \leq z \leq 6 - r$.

Tasks

1. Determine whether the vertical bounds are consistent everywhere in the stated radial range.
2. Correct the radial upper bound if necessary.
3. Give a precise final description and explain which constraint is active.

Question 10 – Solution

Strategy. Vertical consistency requires lower height not exceed upper height.

**Step 1: Compare heights**

$$r^2 \leq 6 - r \iff r^2 + r - 6 \leq 0 \iff (r + 3)(r - 2) \leq 0.$$

With $r \geq 0$, this requires $0 \leq r \leq 2$.

Step 2: Combine radial restrictions The stated planar bound is $r \leq 2 + \cos \theta$, which ranges from 1 to 3. Both conditions must hold, so

$$0 \leq r \leq \min\{2, 2 + \cos \theta\}.$$

Equivalently, the upper bound is 2 when $\cos \theta \geq 0$ and $2 + \cos \theta$ when $\cos \theta < 0$.

Step 3: Final description

$$0 \leq \theta < 2\pi, \quad 0 \leq r \leq \min\{2, 2 + \cos \theta\}, \quad r^2 \leq z \leq 6 - r.$$

The height surfaces meet at $r = 2$. For angles on the right half-plane the height condition truncates the planar radius; on the left half-plane the limaçon-type boundary is already smaller than 2.