

Question 9

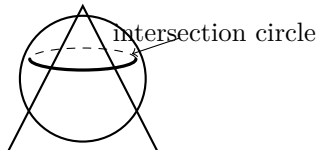
A point has cylindrical coordinates (r, θ, z) with $r = 2z$, $z \geq 0$, and lies on the sphere $x^2 + y^2 + z^2 = 45$.

Tasks

1. Determine r and z .
2. Describe all possible points.
3. Find the horizontal circle containing them.

Question 9 – Solution

Strategy. Convert the sphere to $r^2 + z^2 = 45$ and combine it with the cone relation.



Step 1: Solve Substitute $r = 2z$:

$$(2z)^2 + z^2 = 45 \implies 5z^2 = 45.$$

Since $z \geq 0$, $\boxed{z = 3}$ and $\boxed{r = 6}$.

Step 2: Angular freedom No condition restricts θ , so

$$\boxed{(r, \theta, z) = (6, \theta, 3), \quad 0 \leq \theta < 2\pi}.$$

In Cartesian form the points are $(6 \cos \theta, 6 \sin \theta, 3)$.

Step 3: Locus They form the horizontal circle

$$\boxed{x^2 + y^2 = 36, \quad z = 3}.$$

Verification gives $36 + 9 = 45$ and $r = 6 = 2(3)$.