

Question 8

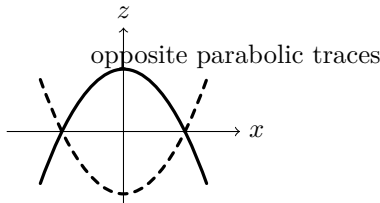
Analyze the surface $z = r^2 \cos 2\theta$.

Tasks

1. Convert it to Cartesian form.
2. Identify its principal vertical traces.
3. Describe its signs by angular sector and its symmetry.

Question 8 – Solution

Strategy. Use $r^2 \cos 2\theta = x^2 - y^2$.



Step 1: Convert Since $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$,

$$r^2 \cos 2\theta = r^2 \cos^2 \theta - r^2 \sin^2 \theta = x^2 - y^2.$$

Thus $\boxed{z = x^2 - y^2}$, a hyperbolic paraboloid.

Step 2: Traces In $y = 0$, $z = x^2$ opens upward. In $x = 0$, $z = -y^2$ opens downward. On $y = \pm x$, $z = 0$.

Step 3: Angular behavior The height is positive where $\cos 2\theta > 0$, negative where it is negative, and zero at $\theta = \pi/4 + k\pi/2$. The equation is unchanged by $\theta \mapsto \theta + \pi$ and by reflections across either coordinate axis, matching the Cartesian symmetries.