

Question 7

Find the distance between cylindrical points $P = (2, \pi/6, 1)$ and $Q = (5, 5\pi/6, 4)$ without first listing both Cartesian points.

Tasks

1. Derive a cylindrical distance formula.
2. Evaluate it exactly.
3. Check the horizontal contribution geometrically.

Question 7 – Solution

Strategy. Expand $(x_1 - x_2)^2 + (y_1 - y_2)^2$ and use the cosine difference identity.

Step 1: Formula For cylindrical points,

$$d^2 = r_1^2 + r_2^2 - 2r_1r_2 \cos(\theta_1 - \theta_2) + (z_1 - z_2)^2.$$

This follows from the planar law of cosines plus the vertical difference.

Step 2: Substitute Here $\theta_1 - \theta_2 = -2\pi/3$, so its cosine is $-1/2$. Thus

$$\begin{aligned} d^2 &= 2^2 + 5^2 - 2(2)(5)(-1/2) + (1 - 4)^2 \\ &= 4 + 25 + 10 + 9 = 48. \end{aligned}$$

Therefore $\boxed{d = 4\sqrt{3}}$.

Step 3: Check The horizontal squared distance is $4 + 25 + 10 = 39$; adding vertical squared distance 9 gives 48, consistent with perpendicular horizontal and vertical components.