

Question 6

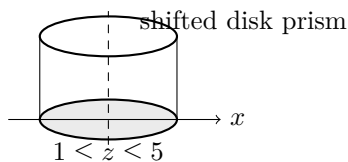
Convert the region $x^2 + y^2 \leq 4x$, $1 \leq z \leq 5$ to cylindrical inequalities.

Tasks

1. Determine the correct angular interval.
2. Give radial and vertical bounds.
3. Explain the geometry and why negative radial bounds are unnecessary.

Question 6 – Solution

Strategy. Convert the disk inequality and enforce the convention $r \geq 0$.



Step 1: Convert The inequality becomes

$$r^2 \leq 4r \cos \theta.$$

With $r \geq 0$, this is $0 \leq r \leq 4 \cos \theta$. The upper bound must be nonnegative, so $\cos \theta \geq 0$.

Step 2: Bounds One nonoverlapping description is

$$\boxed{-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}, \quad 0 \leq r \leq 4 \cos \theta, \quad 1 \leq z \leq 5}.$$

Step 3: Geometry Completing the square gives $(x - 2)^2 + y^2 \leq 4$: a radius-2 disk centered at $(2, 0)$, extended from height 1 to 5. Negative r is unnecessary because directions with $\cos \theta < 0$ would duplicate points already represented using nonnegative r and opposite angles.