

Question 5

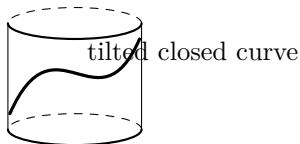
Find a cylindrical parametrization of the curve where $x^2 + y^2 = 9$ and $z = x + y$.

Tasks

1. Express both surfaces cylindrically.
2. Parametrize the entire intersection once.
3. Find its highest and lowest points.

Question 5 – Solution

Strategy. The cylinder fixes r ; let θ be the curve parameter and use the plane for z .



Step 1: Constraints The cylinder gives $r = 3$. The plane becomes

$$z = r \cos \theta + r \sin \theta = 3(\cos \theta + \sin \theta).$$

Thus

$$\mathbf{r}(\theta) = \langle 3 \cos \theta, 3 \sin \theta, 3(\cos \theta + \sin \theta) \rangle, \quad 0 \leq \theta < 2\pi.$$

Step 2: Extrema Use $\cos \theta + \sin \theta = \sqrt{2} \cos(\theta - \pi/4)$. Hence $z_{\max} = 3\sqrt{2}$ at $\theta = \pi/4$, and $z_{\min} = -3\sqrt{2}$ at $\theta = 5\pi/4$.

Step 3: Points

$$P_{\max} = (3/\sqrt{2}, 3/\sqrt{2}, 3\sqrt{2}), \quad P_{\min} = (-3/\sqrt{2}, -3/\sqrt{2}, -3\sqrt{2}).$$

Both satisfy the original cylinder and plane equations.