

Question 3

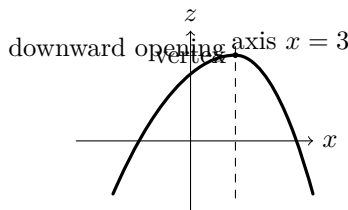
Rewrite $x^2 + y^2 - 6x + 4z = 3$ in cylindrical coordinates and identify the surface.

Tasks

1. Convert and solve for z .
2. Describe horizontal traces.
3. Locate the vertex and axis.

Question 3 – Solution

Strategy. Replace $x^2 + y^2$ by r^2 and x by $r \cos \theta$.



Step 1: Convert

$$r^2 - 6r \cos \theta + 4z = 3, \quad \boxed{z = \frac{3 - r^2 + 6r \cos \theta}{4}}.$$

For geometric identification, complete the Cartesian square:

$$(x - 3)^2 + y^2 = 12 - 4z.$$

Step 2: Identify Equivalently,

$$z = 3 - \frac{(x - 3)^2 + y^2}{4}.$$

This is a downward-opening circular paraboloid with vertical axis $x = 3, y = 0$ and vertex $\boxed{(3, 0, 3)}$.

Step 3: Traces At height $z = c$, the trace is a circle centered at $(3, 0, c)$ with radius $2\sqrt{3 - c}$, present for $c \leq 3$.