

Question 9

A particle follows

$$\mathbf{r}(t) = \langle 3 \cos t, 3 \sin t, 4t \rangle, \quad 0 \leq t \leq 2\pi.$$

Tasks

1. Find its displacement and average velocity over the interval.
2. Find total distance traveled and average speed.
3. Explain why the horizontal motion contributes to distance but not displacement.

Question 9 – Solution

Strategy. Displacement uses endpoints; distance integrates speed. These measure different features of motion.

Step 1: Displacement

$$\mathbf{r}(0) = \langle 3, 0, 0 \rangle, \quad \mathbf{r}(2\pi) = \langle 3, 0, 8\pi \rangle.$$

Thus

$$\Delta \mathbf{r} = \langle 0, 0, 8\pi \rangle.$$

The elapsed time is 2π , so

$$\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{r}}{2\pi} = \langle 0, 0, 4 \rangle.$$

Step 2: Distance and average speed

$$\mathbf{v}(t) = \langle -3 \sin t, 3 \cos t, 4 \rangle,$$

so $\|\mathbf{v}(t)\| = \sqrt{9 + 16} = 5$. Hence

$$D = \int_0^{2\pi} 5 \, dt = 10\pi,$$

and

$$\text{average speed} = \frac{10\pi}{2\pi} = 5.$$

Step 3: Interpretation During one revolution, the horizontal coordinates return to their starting values, so their net changes cancel in displacement. Nevertheless the particle continuously moves horizontally around the circle, adding positive length to the traveled path. Therefore distance 10π exceeds displacement magnitude 8π .