

Question 8

A particle starts at $P = (1, 0, 2)$ with velocity $\langle 2, -1, 0 \rangle$ and constant acceleration $\langle -1, 2, 3 \rangle$.

Tasks

1. Find position and velocity at time t .
2. Determine when the particle first reaches the plane $z = 8$.
3. Find its position and speed then.

Question 8 – Solution

Strategy. For constant acceleration, use $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}_0 + \frac{1}{2}t^2\mathbf{a}$.

Step 1: Motion functions

$$\boxed{\mathbf{v}(t) = \langle 2 - t, -1 + 2t, 3t \rangle},$$

and

$$\boxed{\mathbf{r}(t) = \langle 1 + 2t - \frac{1}{2}t^2, -t + t^2, 2 + \frac{3}{2}t^2 \rangle}.$$

At $t = 0$, these reproduce both initial vectors.

Step 2: Reach $z = 8$

$$2 + \frac{3}{2}t^2 = 8 \quad \implies \quad t^2 = 4.$$

For forward time, $\boxed{t = 2}$.

Step 3: Position and speed

$$\mathbf{r}(2) = \langle 1 + 4 - 2, -2 + 4, 8 \rangle = \boxed{\langle 3, 2, 8 \rangle}.$$

Also

$$\mathbf{v}(2) = \langle 0, 3, 6 \rangle,$$

so

$$\boxed{\text{speed} = \sqrt{0^2 + 3^2 + 6^2} = 3\sqrt{5}}.$$

The vertical coordinate and positive time verify that this is the first forward-time crossing.