

Question 7

A particle has velocity

$$\mathbf{v}(t) = \langle t - 2, 2t + 1, 2 \rangle, \quad t \geq 0.$$

Tasks

1. Find when its speed is smallest.
2. Find the minimum speed.
3. Show that velocity is perpendicular to acceleration at that instant and explain why.

Question 7 – Solution

Strategy. Minimize squared speed, which has the same minimizers as speed and avoids a square root.

Step 1: Squared speed

$$\begin{aligned}q(t) = \|\mathbf{v}(t)\|^2 &= (t-2)^2 + (2t+1)^2 + 4 \\&= 5t^2 + 9.\end{aligned}$$

On $t \geq 0$, this is minimized at $\boxed{t = 0}$.

Step 2: Minimum speed

$$\mathbf{v}(0) = \langle -2, 1, 2 \rangle, \quad \boxed{\|\mathbf{v}(0)\| = 3}.$$

Step 3: Orthogonality Acceleration is

$$\mathbf{a}(t) = \mathbf{v}'(t) = \langle 1, 2, 0 \rangle.$$

At $t = 0$,

$$\mathbf{v}(0) \cdot \mathbf{a}(0) = -2 + 2 + 0 = 0.$$

Also $q'(t) = 2\mathbf{v} \cdot \mathbf{a} = 10t$, so the vanishing dot product is exactly the zero derivative of squared speed. Here the minimum occurs at the endpoint, and the right-hand derivative is nonnegative immediately afterward.