

Question 6

Two objects move according to

$$\mathbf{r}_1(t) = \langle t, 2t, 3 - t \rangle, \quad \mathbf{r}_2(t) = \langle 4 - t, t + 1, 2t - 2 \rangle, \quad t \geq 0.$$

Tasks

1. Determine whether they collide.
2. Find their velocities and relative velocity.
3. Verify your collision conclusion using relative position.

Question 6 – Solution

Strategy. A collision requires equal position vectors at the same time, not merely intersecting geometric paths.

Step 1: Match coordinates Equality requires

$$t = 4 - t, \quad 2t = t + 1, \quad 3 - t = 2t - 2.$$

The first gives $t = 2$, while the second gives $t = 1$. Since no single time satisfies even these two equations, the objects do not collide:

$$\boxed{\text{no collision for } t \geq 0}.$$

Step 2: Velocities

$$\mathbf{v}_1 = \langle 1, 2, -1 \rangle, \quad \mathbf{v}_2 = \langle -1, 1, 2 \rangle.$$

The velocity of object 1 relative to object 2 is

$$\boxed{\mathbf{v}_{1/2} = \mathbf{v}_1 - \mathbf{v}_2 = \langle 2, 1, -3 \rangle}.$$

Step 3: Relative-position check

$$\mathbf{r}_1 - \mathbf{r}_2 = \langle 2t - 4, t - 1, 5 - 3t \rangle.$$

For this vector to be zero, its first, second, and third components would require respectively $t = 2$, $t = 1$, and $t = 5/3$. The inconsistent times independently confirm that relative position never vanishes.