

Question 5

For the helical motion

$$\mathbf{r}(t) = \langle 3 \cos 2t, 3 \sin 2t, 4t \rangle,$$

determine the velocity, acceleration, speed, and angle between velocity and the positive z -axis.

Tasks

1. Compute $\mathbf{v}$ and $\mathbf{a}$.
2. Show speed and the requested angle are constant.
3. Prove acceleration points horizontally toward the helix axis.

Question 5 – Solution

Strategy. Differentiate, then use the dot-product angle formula with $\mathbf{k} = \langle 0, 0, 1 \rangle$.

Step 1: Motion quantities

$$\mathbf{v}(t) = \langle -6 \sin 2t, 6 \cos 2t, 4 \rangle,$$

and

$$\mathbf{a}(t) = \langle -12 \cos 2t, -12 \sin 2t, 0 \rangle.$$

The speed is

$$\|\mathbf{v}\| = \sqrt{36(\sin^2 2t + \cos^2 2t) + 16} = 2\sqrt{13}.$$

Step 2: Direction angle If θ is the angle with the positive z -axis,

$$\cos \theta = \frac{\mathbf{v} \cdot \mathbf{k}}{\|\mathbf{v}\| \|\mathbf{k}\|} = \frac{4}{2\sqrt{13}} = \frac{2}{\sqrt{13}}.$$

Therefore

$$\theta = \cos^{-1} \left(\frac{2}{\sqrt{13}} \right).$$

No t occurs, so both speed and angle are constant.

Step 3: Acceleration geometry Its horizontal component is

$$-4 \langle 3 \cos 2t, 3 \sin 2t, 0 \rangle,$$

which is opposite the radial position from the z -axis. Its z -component is zero, so acceleration points horizontally inward. Also $\mathbf{v} \cdot \mathbf{a} = 0$, consistent with constant speed.