

Question 4

A particle moves on a circle according to

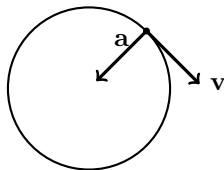
$$\mathbf{r}(t) = \langle R \cos(\omega t), R \sin(\omega t), 0 \rangle, \quad R > 0, \omega > 0.$$

Tasks

1. Find velocity, speed, and acceleration.
2. Prove velocity is perpendicular to acceleration.
3. Express acceleration in terms of position and identify its magnitude.

Question 4 – Solution

Strategy. Differentiate symbolically and use trigonometric identities and dot products.



velocity tangent; acceleration inward

Step 1: Differentiate

$$\mathbf{v} = \langle -R\omega \sin(\omega t), R\omega \cos(\omega t), 0 \rangle,$$

so

$$\|\mathbf{v}\| = R\omega.$$

Differentiating again,

$$\mathbf{a} = \langle -R\omega^2 \cos(\omega t), -R\omega^2 \sin(\omega t), 0 \rangle.$$

Step 2: Orthogonality

$$\mathbf{v} \cdot \mathbf{a} = R^2\omega^3 \sin(\omega t) \cos(\omega t) - R^2\omega^3 \cos(\omega t) \sin(\omega t) = 0.$$

Thus $\mathbf{v} \perp \mathbf{a}$.

Step 3: Inward acceleration Comparing with $\mathbf{r}$ gives

$$\mathbf{a}(t) = -\omega^2 \mathbf{r}(t).$$

It points toward the origin, and

$$\|\mathbf{a}\| = R\omega^2 = \frac{(R\omega)^2}{R} = \frac{v^2}{R}.$$

This is purely normal acceleration; the constant speed confirms there is no tangential component.