

Question 3

A particle satisfies

$$\mathbf{a}(t) = \langle 6t, 4e^{2t}, -2 \rangle, \quad \mathbf{v}(0) = \langle 1, -1, 3 \rangle, \quad \mathbf{r}(0) = \langle 2, 0, -4 \rangle.$$

Tasks

1. Recover $\mathbf{v}(t)$.
2. Recover $\mathbf{r}(t)$.
3. Verify both initial conditions and the acceleration.

Question 3 – Solution

Strategy. Integrate componentwise twice, determining a new constant vector at each stage.

Step 1: Velocity

$$\mathbf{v}(t) = \langle 3t^2 + C_1, 2e^{2t} + C_2, -2t + C_3 \rangle.$$

At $t = 0$, comparison with $\langle 1, -1, 3 \rangle$ gives

$$C_1 = 1, \quad 2 + C_2 = -1, \quad C_3 = 3.$$

Therefore

$$\boxed{\mathbf{v}(t) = \langle 3t^2 + 1, 2e^{2t} - 3, 3 - 2t \rangle}.$$

Step 2: Position Integrating velocity,

$$\mathbf{r}(t) = \langle t^3 + t + D_1, e^{2t} - 3t + D_2, 3t - t^2 + D_3 \rangle.$$

Using $\mathbf{r}(0) = \langle 2, 0, -4 \rangle$ gives $D_1 = 2$, $1 + D_2 = 0$, and $D_3 = -4$. Hence

$$\boxed{\mathbf{r}(t) = \langle t^3 + t + 2, e^{2t} - 3t - 1, 3t - t^2 - 4 \rangle}.$$

Step 3: Verification Differentiating $\mathbf{r}$ gives the displayed $\mathbf{v}$; differentiating again gives $\langle 6t, 4e^{2t}, -2 \rangle$. Direct substitution at $t = 0$ returns both required initial vectors.