

Question 2

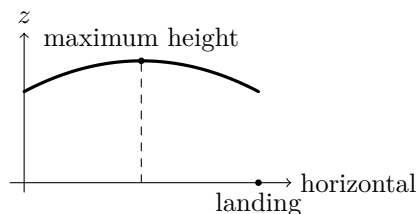
A ball is launched from $(0, 0, 5)$ with initial velocity $\langle 30, 40, 50 \rangle$ ft/s. Gravity is $\langle 0, 0, -32 \rangle$ ft/s² and air resistance is neglected.

Tasks

1. Find position and velocity for $t \geq 0$.
2. Find the maximum height and when it occurs.
3. Find the landing time and horizontal landing point.

Question 2 – Solution

Strategy. Integrate constant acceleration twice and apply the initial data; landing occurs when the vertical coordinate is zero.



Step 1: Motion functions

$$\mathbf{v}(t) = \langle 30, 40, 50 - 32t \rangle.$$

Integrating again and using $\mathbf{r}(0) = \langle 0, 0, 5 \rangle$,

$$\mathbf{r}(t) = \langle 30t, 40t, 5 + 50t - 16t^2 \rangle.$$

Step 2: Maximum height

The vertical velocity is zero when

$$50 - 32t = 0 \implies t = \frac{25}{16}.$$

Then

$$z = 5 + 50\left(\frac{25}{16}\right) - 16\left(\frac{25}{16}\right)^2 = \boxed{\frac{705}{16} \text{ ft}}.$$

Step 3: Landing

Set $z = 0$:

$$5 + 50t - 16t^2 = 0 \implies t = \frac{25 \pm \sqrt{705}}{16}.$$

Only the positive root is physical:

$$t_L = \frac{25 + \sqrt{705}}{16}.$$

Thus the horizontal landing point is

$$(x, y) = (30t_L, 40t_L).$$

Substitution into the vertical equation verifies height zero.