

**Question 1**

A particle has position

$$\mathbf{r}(t) = \langle t^2 - 2t, t^3 - 3t, 4t + 1 \rangle, \quad t \geq 0.$$

**Tasks**

1. Find velocity, acceleration, and speed.
2. Evaluate them at  $t = 1$ .
3. Find the tangent line to the trajectory at that time.

**Question 1 – Solution**

**Strategy.** Differentiate position componentwise; speed is the magnitude of velocity.

**Step 1: Velocity and acceleration**

$$\mathbf{v}(t) = \mathbf{r}'(t) = \langle 2t - 2, 3t^2 - 3, 4 \rangle,$$

and

$$\mathbf{a}(t) = \mathbf{v}'(t) = \langle 2, 6t, 0 \rangle.$$

Therefore

$$\text{speed} = \|\mathbf{v}(t)\| = \sqrt{(2t - 2)^2 + (3t^2 - 3)^2 + 16}.$$

**Step 2: Values at  $t = 1$**

$$\mathbf{r}(1) = \langle -1, -2, 5 \rangle, \quad \mathbf{v}(1) = \langle 0, 0, 4 \rangle, \quad \mathbf{a}(1) = \langle 2, 6, 0 \rangle.$$

The speed is  $\boxed{4}$ .

**Step 3: Tangent line** Since  $\mathbf{v}(1) \neq \mathbf{0}$ , it gives the tangent direction:

$$\mathbf{L}(s) = \langle -1, -2, 5 \rangle + s \langle 0, 0, 1 \rangle.$$

The simplified direction is parallel to velocity. At  $s = 0$  the line passes through the correct position.