

Question 9

A helix must have radius 3 and constant curvature $1/5$. It is modeled by

$$\mathbf{r}(t) = \langle 3 \cos t, 3 \sin t, bt \rangle, \quad b > 0.$$

Tasks

1. Determine b exactly.
2. Find the vertical rise per complete revolution.
3. Verify the designed curvature directly.

Question 9 – Solution

Strategy. Use the helix curvature formula and then translate parameter change into vertical rise.

Step 1: Solve the design equation For a helix of radius a ,

$$\kappa = \frac{a}{a^2 + b^2}.$$

With $a = 3$ and $\kappa = 1/5$,

$$\begin{aligned}\frac{3}{9 + b^2} &= \frac{1}{5}, \\ 15 &= 9 + b^2, \\ b^2 &= 6.\end{aligned}$$

Since $b > 0$, $\boxed{b = \sqrt{6}}$.

Step 2: Rise per revolution A full revolution changes t by 2π . Since $z = bt$,

$$\Delta z = b(2\pi) = \boxed{2\pi\sqrt{6}}.$$

Step 3: Verification Substituting $b^2 = 6$ into the curvature formula gives

$$\kappa = \frac{3}{3^2 + (\sqrt{6})^2} = \frac{3}{9 + 6} = \boxed{\frac{1}{5}}.$$

The positive root also makes the helix rise as t increases, satisfying the stated orientation.