

Question 6

At a point of a regular curve,

$$\mathbf{r}' = \langle 2, -1, 2 \rangle, \quad \mathbf{r}'' = \langle 1, 2, 0 \rangle.$$

Tasks

1. Compute the curvature.
2. Decompose $\mathbf{r}''$ into components parallel and perpendicular to $\mathbf{r}'$.
3. Verify curvature using $\kappa = \|\mathbf{r}''_{\perp}\|/\|\mathbf{r}'\|^2$.

Question 6 – Solution

Strategy. Compare the cross-product formula with the geometric perpendicular-acceleration formula.

Step 1: Cross-product computation

$$\mathbf{r}' \times \mathbf{r}'' = \langle 2, -1, 2 \rangle \times \langle 1, 2, 0 \rangle = \langle -4, 2, 5 \rangle.$$

Its length is $\sqrt{16 + 4 + 25} = 3\sqrt{5}$, while $\|\mathbf{r}'\| = 3$. Therefore

$$\kappa = \frac{3\sqrt{5}}{3^3} = \frac{\sqrt{5}}{9}.$$

Step 2: Parallel component Since

$$\mathbf{r}'' \cdot \mathbf{r}' = 2 - 2 + 0 = 0,$$

the projection of $\mathbf{r}''$ onto $\mathbf{r}'$ is

$$\mathbf{r}''_{\parallel} = \frac{\mathbf{r}'' \cdot \mathbf{r}'}{\|\mathbf{r}'\|^2} \mathbf{r}' = \mathbf{0}.$$

Thus $\mathbf{r}''_{\perp} = \mathbf{r}'' = \langle 1, 2, 0 \rangle$ and $\|\mathbf{r}''_{\perp}\| = \sqrt{5}$.

Step 3: Verification

$$\frac{\|\mathbf{r}''_{\perp}\|}{\|\mathbf{r}'\|^2} = \frac{\sqrt{5}}{9},$$

which agrees exactly with the cross-product result.