

Question 10

A proposed sphere is required to satisfy all three conditions below:

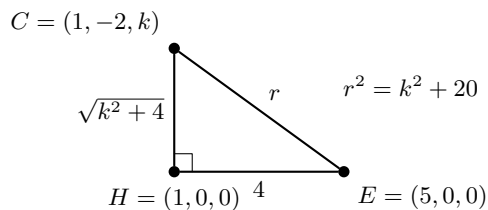
1. It intersects the x -axis at $A = (-3, 0, 0)$ and $B = (5, 0, 0)$.
2. Its center lies in the plane $y = -2$.
3. It is tangent to the xy -plane.

Consistency test

Determine whether such a sphere exists. If it does, find every possible center, radius, and equation. If it does not, prove precisely which requirements are incompatible.

Question 10 – Solution

Strategy The center lies on the perpendicular-bisector plane of the two axial intersection points. Tangency to the xy -plane relates the radius to the center's height.



Center coordinates The midpoint of the two intersections has x -coordinate 1, so write the center as

$$C = (1, -2, k).$$

The squared radius, using $(5, 0, 0)$, is

$$r^2 = (5 - 1)^2 + (0 + 2)^2 + k^2 = 20 + k^2.$$

Tangency to the xy -plane requires $r = |k|$, hence $r^2 = k^2$. Combining these equations would give $k^2 = 20 + k^2$, an impossibility.

Conclusion Therefore no sphere satisfies all the stated conditions.

Consistency review The two axial intersection points already force every possible center onto $x = 1$. Requiring $y = -2$ means the center has a nonzero horizontal offset from the x -axis. Consequently the radius needed to reach either axial point is strictly greater than $|k|$, while tangency to the xy -plane requires it to equal $|k|$. The contradiction is geometric, not an algebraic accident.