

Question 9

A rectangular box has edges parallel to the coordinate axes and opposite vertices

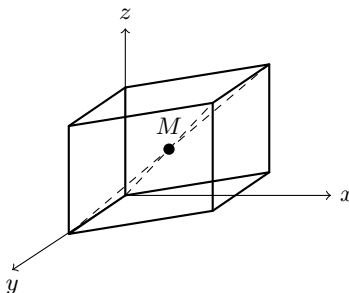
$$A = (1, -2, 3), \quad G = (7, 4, 13).$$

Tasks

1. List all eight vertices systematically.
2. Show that the four space diagonals have a common midpoint, and find it.
3. Find the center and radius of the sphere passing through all eight vertices.
4. Verify that every vertex lies on this sphere without substituting the vertices one at a time.

Question 9 – Solution

Strategy Each vertex independently chooses one of the two available values for each coordinate. Opposite vertices choose complementary values.



Vertices The coordinate choices are $x \in \{1, 7\}$, $y \in \{-2, 4\}$, $z \in \{3, 13\}$. Thus the vertices are

$$(1, -2, 3), (1, -2, 13), (1, 4, 3), (1, 4, 13),$$

$$(7, -2, 3), (7, -2, 13), (7, 4, 3), (7, 4, 13).$$

Common midpoint Every opposite pair averages the low and high value in each coordinate, giving

$$M = \left(\frac{1+7}{2}, \frac{-2+4}{2}, \frac{3+13}{2} \right) = \boxed{(4, 1, 8)}.$$

Hence all four space diagonals bisect one another at M .

Circumsphere From M to any vertex, the coordinate changes have magnitudes 3, 3, 5. Therefore

$$\boxed{r = \sqrt{3^2 + 3^2 + 5^2} = \sqrt{43}}.$$

Verification Every vertex has one of the sign patterns $(\pm 3, \pm 3, \pm 5)$ relative to M , so all eight lie on the sphere.